INVESTIGATING GRAPHS USING DESMOS.COM

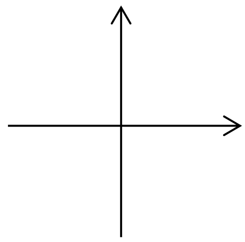
Desmos is a free graphing website and app that is extremely useful and highly recommended for use at A-Level

Section A: Sketching Quadratic functions

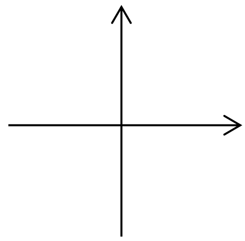
Quadratics have an equation of the form $y = ax^2 + bx + c$ where *a, b* & *c* are constants & $a \neq 0$

Activity 1 – Positive and negative quadratic functions

1. Plot on Desmos the quadratic function $y = x^2 + 4x + 5$ and sketch the general shape on the axes opposite



2. Plot on Desmos the quadratic function $y = -x^2 - x + 2$ and sketch the general shape on the axes
opposite



3. By assessing the shape of the quadratic complete the below statements:

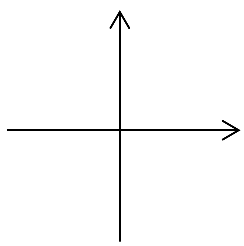
When $a > 0$ the shape of the curve will be:

When $a < 0$ the shape of the curve will be:

The shape of the graph is called a ***parabola***.

Activity 2 – finding out where the quadratic curve cuts the coordinate axes

1. Plot on Desmos the quadratic curve $y = x^2 + 2x - 3$ and sketch it opposite showing where the curve cuts the coordinate axes



2. Solve the quadratic $x^2 + 2x - 3 = 0$ ……………………………………………………………………………

3. Write down the coordinates where the curve cuts the *x*-axis…………………………………………

4. Write down the coordinates where the curve cuts the *y*-axis…………………………………………

5. Write a brief statement on how you find where the curve cuts the *y*-axis (ensure you mention what *x* is equated to)
………

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6. Write a brief statement on how you find where the curve cuts the *x*-axis (ensure you mention what *y* is equated to)
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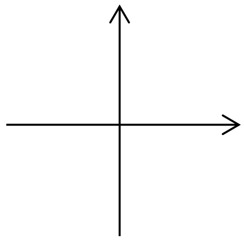
Activity 3 – finding turning points



1. Plot on Desmos the quadratic curve $y = x^2 + 2x + 3$and sketch it opposite showing any key coordinates.

2. What do you notice about the curve and the *x*-*a*xis?

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3. Write down the coordinates of the minimum point on the curve...

4. Complete the square (but do not solve) for the quadratic$x^2 + 2x + 3$leaving it in the form $(x + p)^2 + q$

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5. What do you notice about the minimum point and the numbers *p* & *q*? Write a brief statement on how to find turning points.

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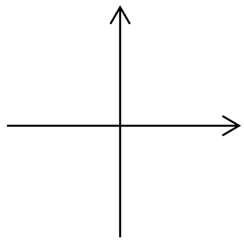
Activity 4 – Sketching quadratics and the discriminant

DISCRIMINANT: $b^2 - 4ac$ **This is part that is beneath the square root in the quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\text{when } ax^2+bx+c=0$$

Working out the discriminant before sketching a quadratic can be very handy.



1. a) Plot on Desmos the function $y = x^2 - 6x + 5$and sketch it opposite.

b) How many solutions are there to the equation $x^2 - 6x + 5 = 0$ (hint: how many times does the curve cross the *x*-axis?).

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c) Work out $b^2 - 4ac$

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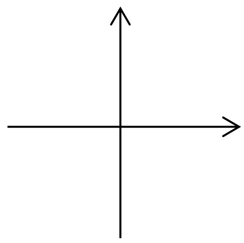

2. a) Plot on Demsos the function $y = x^2 + 6x + 9$and sketch it opposite.

b) How many solutions are there to the equation $x^2 + 6x + 9 = 0$ (hint: how many times does the curve cross the *x*-axis?).

....

c) Work out $b^2 - 4ac$

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....

3. a) Plot on Desmos the function $y = x^2 + 2x + 3$and sketch it opposite.

b) How many solutions are there to the equation $x^2 + 2x + 3 = 0$ (hint: how many times does the curve cross the *x*-axis?).

………

c) Work out $b^2 - 4ac$

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Using your answers to q1-3 write down how many solutions (or roots) there are to a quadratic when the discriminant is positive, zero or negative (remember that “roots” means how many times the quadratic touces or crosses the *x*-axis).

$b^2 - 4ac > 0$ ……

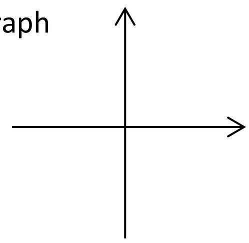
$b^2 - 4ac = 0$ ……

$b^2 - 4ac < 0$ ……

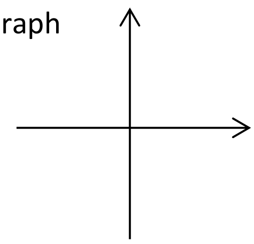
<u>Section B: Sketching cubic functions</u>

Cubics have an equation of the form $y = ax^3 + bx^2 + cx + d$ where *a, b, c* & *d* are constants & $a \neq 0$

1. Plot on Desmos the cubic function $y = x^3 - 2x^2 - 3x + 2$ and sketch the shape of the graph opposite



2. Plot on Desmos the cubic function $y = -x^3 + 2x^2 + x - 2$ and sketch the shape of the graph opposite



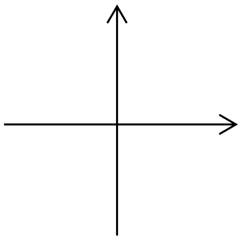
3. By assessing the shape of the cubic complete the below statements:

When $a > 0$ the shape of the curve will be:

When $a < 0$ the shape of the curve will be:

Activity 2 – finding out where the cubic curve cuts the coordinate axes

1. Plot on Desmos the cubic function $y = x^3 - 4x^2 + x + 6$and sketch it opposite showing where the curve cuts the coordinate axes



You will be asked to factorise a cubic of the form$y = ax^3 + bx^2 + cx + d$, but for the purposes of this exercise, you will be given it already factorised.

The cubic $x^3 - 4x^2 + x + 6$factorises to $(x-2)(x-3)(x+1)$

2. Write down the coordinates where the curve cuts the *x*-axis…………………………………………

3. Write down the coordinates where the curve cuts the *y*-axis…………………………………………

4. Write a brief statement on how you find where the curve cuts the *y*-axis

………

………

5. Write a brief statement on how you find where the curve cuts the *x*-axis

………

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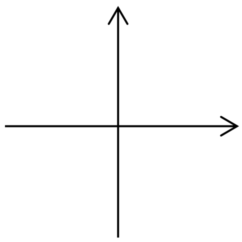
Activity 3 – cubic functions of the form $y = ax^3 + bx^2 + cx$

You may be given a cubic function of the form $y = ax^3 + bx^2 + cx$.

With this type you can factorise easily and subsequently find where the curve cuts the axes.



1. By first factorising out an *x*, factorise $y = x^3 + 3x^2 + 2x$ and solve (equate *y* to 0)
Now sketch the curve opposite.
How many solutions are there to $x^3 + 3x^2 + 2x = 0$ (i.e. how many times does it cross the *x*-axis?)

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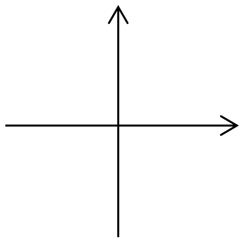
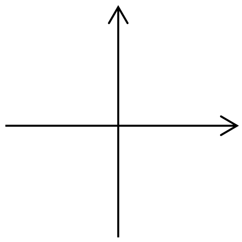
NOTE: Indicating where the turning points are located is not required at this stage, but it will be something that you will have to do.

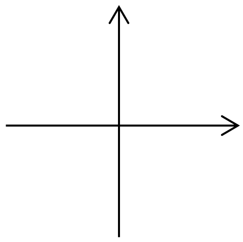
Activity 4 – different types of cubic



1. Plot on Desmos $y = x(x-2)^2$ and sketch it opposite.
Note that this cubic could be written as $y = x(x-2)(x-2)$. In which case there is a "repeated root" and one other root.
How many solutions are there to $x(x-2)^2 = 0$ (i.e. how many times does it cross the *x*-axis?)
Make a brief comment about what happens to the graph when *x* = 2.



2. Plot on Desmos $y = (x-4)^3$ and sketch it opposite.
Again, this cubic could be written as $y = (x-4)(x-4)(x-4)$.
How many solutions are there to $(x-4)^3 = 0$ (i.e. how many times does it cross the *x*-axis?)
Make a brief comment about what happens to the graph when *x* = 4.



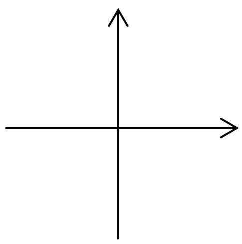
3. Plot on Desmos $y = (x-1)(x^2+x+2)$ and sketch it opposite.
How many solutions are there to $(x-1)(x^2+x+2) = 0$ (i.e. how many times does it cross the *x*-axis?)

Why do you think the answer to the above is so? (Hint: is the quadratic in the second bracket solvable when equated to 0)?

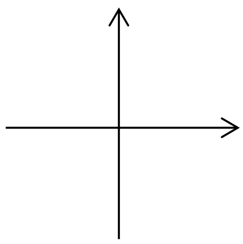
Section C: Sketching quartic functions

Quartics have an equation of the form $y = ax^4 + bx^3 + cx^2 + dx + e$ **where *a, b, c, d* and *e* are constants & $a \neq 0$**

- A positive coefficient of x^4 will mean the graph 'goes up' at both ends
- A negative coefficient of x^4 will mean the graph 'goes down' at both ends



Activity 1
Use Desmos to plot the two curves below and sketch them opposite.
a) $y = (x+1)(x+2)(x-1)(x-2)$
b) $y = x(x+2)^2(3-x)$

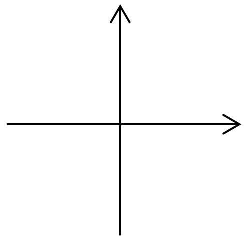


Express these quartics in the form $y = ax^4 + bx^3 + cx^2 + dx + e$

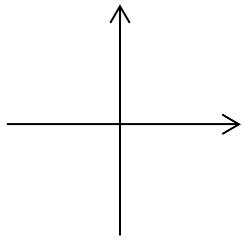
The tails of a quartic will either both go up or both go down.
In between, the graph will have *up to* 3 turning points. There may be as many as 4 roots (where the curve crosses the *x*-axis.)

Activity 2
Use Desmos to plot these quartics and sketch them on the given axes.

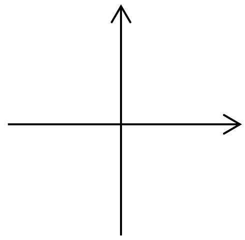
a) $y = (x+3)(x+2)(x-1)(x-3)$
How many distinct roots does it have?
Why do you think this is?



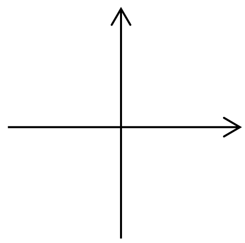
b) $y = x(x-2)^2(4-x)$
How many distinct roots does it have?
What happens at *x* = 2? Why do you think this is?



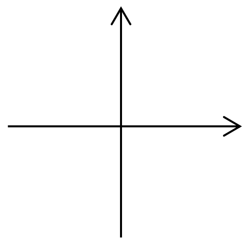
c) $y = (x-1)^2(x-3)^2$
How many distinct roots does it have?
What happens at *x* = 1 and *x* = 3? Why do you think this is?



d) $y = (x-1)^2(x-3)(x+1)$
How many distinct roots does it have?



e) $y = (x-1)^4$
How many distinct roots does it have?

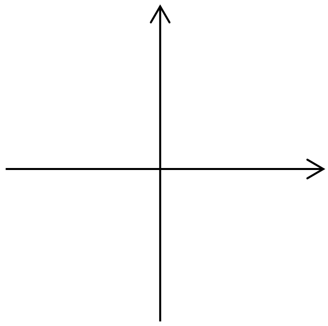


Section D: reciprocal graphs

Reciprocal graphs have a general form $\boldsymbol{y = \frac{a}{x^n}}$

Activity 1

On Desmos plot the curves $y = \frac{1}{x}$, $y = \frac{5}{x}$, $y = \frac{10}{x}$ & $y = -\frac{3}{x}$. Sketch and label the curves on the graph opposite.



What happens to the curve as the constant *a* increases?

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What do you notice about the curve and the *x* axis? Why do you think this happens?

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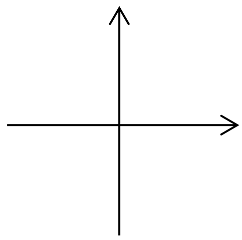
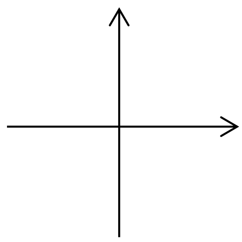
Activity 2



a) On Desmos to plot the curve $y = \frac{2}{x^2}$. Sketch and label opposite.

b) Repeat for the curve $y = -\frac{2}{x^2}$



c) For what values of *x* is this curve similar to that of $y = \frac{2}{x}$?

………

d) For what values of *x* is this curve different to that of $y = \frac{2}{x}$?

………

e) Explain why you think these

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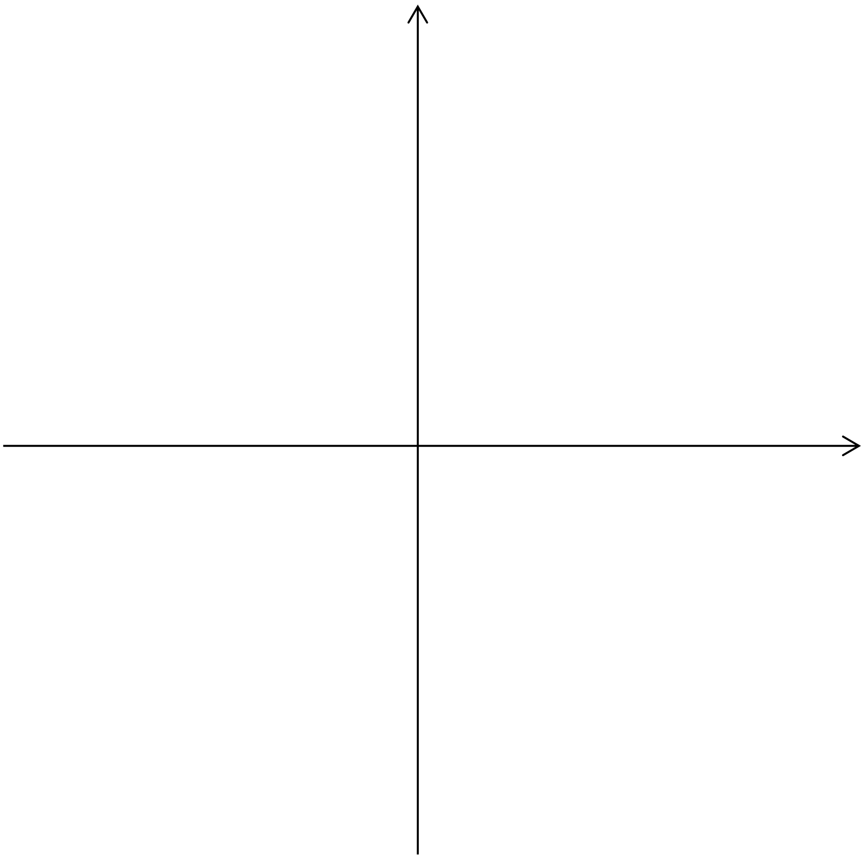
SECTION E: Using sketches to help you solve points of intersection

1. On the same diagram sketch the curves with equations $y = x(x-3)$ & $y = x^2(1-x)$. Try doing this without Desmos and then check your answer using Desmos.

2. How many points of intersection are there?

3. Find the coordinates of intersection and label them on your sketch.

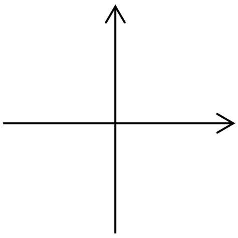


Of course, although graphical calculators are allowed in A-Level exams, you will be asked to find points of intersection using algebraic techniques i.e. solving simultaneous equations.

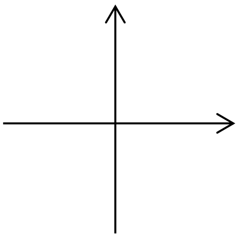
<u>SECTION F: Without using Desmos, sketch the below curves (you may use Desmos to check your answers afterwards)</u>

You need to show on your sketches where the curves cross the coordinate axes (if at all) and show your algebraic method

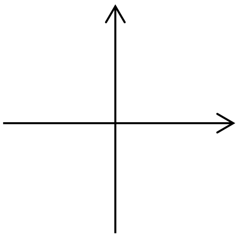
$y = x^2 - 9x + 20$



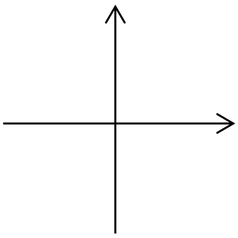
$y = -x^2 + 8x - 15$



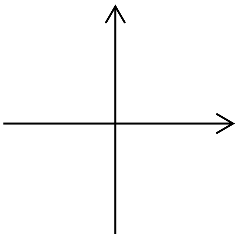
$y = x^3 + 3x^2 + 2x$



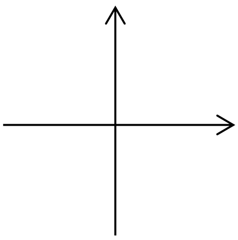
$y = (x - 3)(x + 5)(x + 2)$



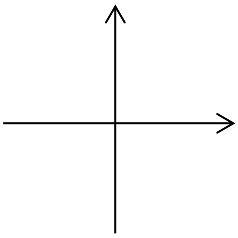
$y = (x - 3)^2(x + 2)$



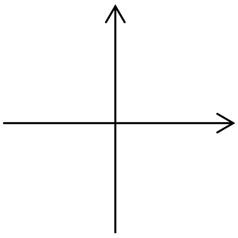
$y = -(x + 5)^3$



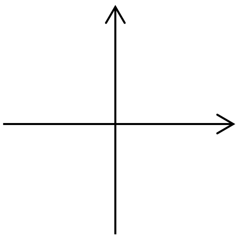
$y = (x-3)(x+5)(x+2)(x+1)$



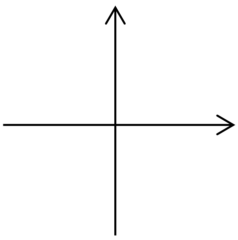
$y = (x-2)^2(x+5)(x+2)$



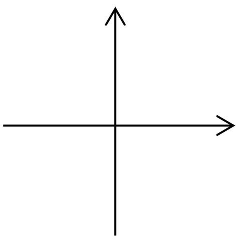
$y = (x-2)^2(x+5)^2$



$y = (x+2)^3(x-1)$



$y = -(x+3)^4$



Sketch the following on the same set of axes and show any intersections that the curves may have. You must show algebraically how you found the points of intersection

$$y = \frac{4}{x^2} \qquad y = \frac{1}{x}$$